Exploration of Key Instructional Advances in Mathematics

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Three Key Advances Prompted by the CCR Standards

1. **Focus**: Focus strongly where the standards focus.
2. **Coherence**: Design learning around coherent progressions level to level.
3. **Rigor**: Pursue conceptual understanding, procedural skill and fluency, and application—all with equal intensity.

Mathematics Advance One: **Focus Strongly Where the CCR Standards Focus**

Implications for Instruction

- Focus on the major work means that some content is more important than other content:
  - Focusing narrows but deepens the scope of content and shows the "power of the eraser."
  - Rather than "a mile wide and an inch deep," focusing results in a "mile deep and an inch wide."
  - Focusing opens the door to strengthening understanding—fewer topics on the list means more time to spend on each one.
- Other content supports the major work of the level.

Major Areas of Focus

- *Level A:* Whole numbers—addition and subtraction concepts, skills, and problem solving to 20; place value and whole number relationships to 100; and reasoning about geometric shapes and linear measures
- *Level B:* Whole numbers and fractions—place value, comparison, and addition and subtraction to 1000, fluency to 100; multiplication and division to 100; fractions concepts, skills, and problem solving; and 2-dimensional shape concepts; standard units and area measurements

Major Areas of Focus

- *Level C:* Whole numbers, fractions, and decimals—fluency with multi-digit whole number and decimal operations; decimal place value concepts and skills to thousandths; comparing, ordering, and operating with fractions; fluency with sums and differences of fractions; understanding rates and ratios; early expressions and equations; the coordinate plane, including creating dot plots from data; area, surface area, and volume; classification of 2-dimensional shapes; and developing understanding of data distributions

Major Areas of Focus

- *Level D:* Rational numbers—fluent arithmetic of positive and negative rational numbers; rates, ratios, and proportions; linear expressions, equations, and functions; classification and analysis of 2- and 3-dimensional figures; similarity and congruence concepts; random sampling of populations to summarize, describe, display, interpret, and draw inferences; and development of probability concepts

Major Areas of Focus

- *Level E:* Real numbers—extending number system to include all real numbers; expressions involving radicals and integer exponents; reasoning with units and levels of precision; linear, quadratic, exponential expressions, equations, and functions; linear inequalities; algebraic and graphic models of functions; right triangle relationships; probability concepts; and one- and two-variable data sets, including frequency tables

Focusing on the Major Work of Each Level

- Purpose: to develop an understanding of Focus
- Materials: "Major Work of the CCR Adult Education Levels" reference sheet and "Focusing on the Major Work of Each Level" worksheet
- Directions: Working with a partner, circle the topics at each level that are the major focus of that level.

Activity

Connect the Standards for Mathematical Practice to Content

Progressions of Domains and MPs

<table>
<tr><th>Level A</th><th>Level B</th><th>Level C</th><th>Level D</th><th>Level E</th></tr>
<tr><td colspan="5">Mathematical Practices
1) Make sense of problems and persevere in solving them.
2) Reason abstractly and quantitatively.
3) Construct viable arguments and critique the reasoning of others.
4) Model with mathematics.
5) Use appropriate tools strategically.
6) Attend to precision.
7) Look for and make use of structure.
8) Look for and express regularity in repeated reasoning.</td></tr>
<tr><td colspan="2">Number and operations: Base Ten (Levels A – C)</td><td colspan="2">Ratios and Proportional relationships (Levels C – D)</td><td rowspan="2">Number and Quantity (Level E) [N.RN; N.Q]</td></tr>
<tr><td></td><td>Number and operations: Fractions (Levels B – C)</td><td colspan="2">the Number System (Levels C – D)</td></tr>
<tr><td colspan="2" rowspan="2">Operations and Algebraic thinking (Levels A – C)</td><td colspan="2">Expressions and Equations (Levels C – D)</td><td>Algebra (Level E) [A.SSE; A.APR; A.CED; A.REI]</td></tr>
<tr><td></td><td>Functions (Level D)</td><td>Functions (Level E) [F.IF; F.BF; F.LE]</td></tr>
<tr><td colspan="4">Geometry (Levels A – D)</td><td>Geometry (Level E) [G.CO; G.SRT; G.GMD; G.MG]</td></tr>
<tr><td colspan="2">Measurement and Data (Levels A – C)</td><td colspan="2">Statistics and Probability (Level C – D)</td><td>Statistics and Probability: Interpreting Categorical and Quantitative Data</td></tr>
</table>

Implications for Instruction

- The Standards for Mathematical Practice are meant to be applied across all levels.
- Not all Standards for Mathematical Practice are appropriate for every lesson—focus should only be on those Practices that are central.
- Important to see to it that there are opportunities to experience *all* the Standards for Mathematical Practice for students over the unit or the level of study.

Standards for Mathematical Practice

MP.1 Make sense of problems and persevere in solving them.

MP.2 Reason abstractly and quantitatively.

MP.3 Construct viable arguments and critique the reasoning of others.

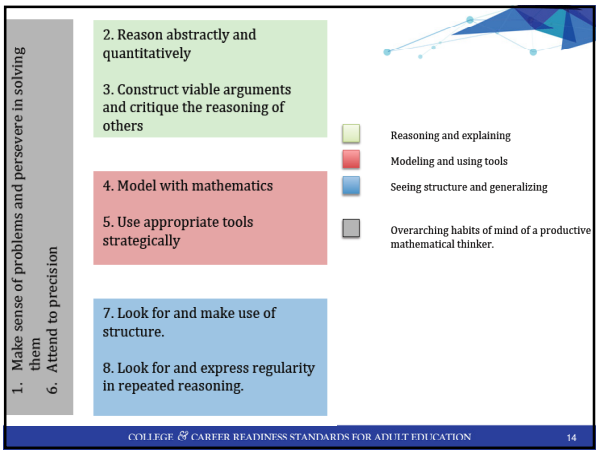
MP.4 Model with mathematics.

MP.5 Use appropriate tools strategically.

MP.6 Attend to precision.

MP.7 Look for and make use of structure.

MP.8 Look for and express regularity in repeated reasoning.



Integrating the Standards for Mathematical Practices Into Lessons (Part 1)

- Purpose: to develop an understanding of the Standards for Mathematical Practice
- Materials: "CCR Standards for Mathematical Practice"
- Directions: Read through the Standards for Mathematical Practice, highlighting characteristics and skills of mathematically proficient students.

Activity

Integrating the Standards for Mathematical Practices Into Lessons (Part 2)

- Purpose: to develop an understanding of the Standards for Mathematical Practice in practice
- Materials: "Enriching a Lesson with Standards for Mathematical Practice" worksheet and *Level C - Equivalent Fractions* sample lesson
- Directions: Scan the lesson and make notes about how MPs might be observed in the activities. Use the X and O as indicated on the worksheet.

Activity

Mathematics Advance Two: **Design Learning Around Coherent Progressions Level to Level**

Implications for Instruction

- Lessons should build new understanding on foundations built in previous lessons or levels so that content unfolds meaningfully.
- Explicit connections between content should be made in lessons across the levels but also within a level.
- Students and teachers should begin to expect knowledge and skills to build and grow: Each standard is *not a new event*, but an *extension of previous learning*.

Thinking Across Levels to Connect Learning

- Purpose: to develop an understanding of Coherence
- Materials: three sets of cards (one each: yellow, green, and blue) and CCR Standards or "Major Work of the CCR Adult Education Levels" reference sheet (optional)
- Directions:

1) Use the language of the standards to identify the progression topic of each set of cards: Fluency, Expressions and Equations, and Application
2) In each progression topic, identify the level (A, B, C, D, E) of the standard.

Activity

Mathematics Advance Three: **Pursue Conceptual Understanding, Procedural Skill and Fluency, and Application—All With Equal Intensity**

Implications for Instruction

Rigor in lessons relates to the depth at which the major work of each level should be addressed:

- Lessons should help students deeply understand key concepts and see math as more than a set of discrete procedures. (conceptual understanding)
- Some class time and homework should be devoted to students' practice with calculations and mathematical procedures so they gain speed and accuracy. (fluency)
- Fluency practice is critical but not an end in itself; rather it is used to support problem solving and deeper mathematical thinking. (application)

Why Are the Three Components Necessary?

- Knowing more than "how to get the answer" allows students to generalize knowledge to new situations and types of problems (conceptual understanding and application).
- Being able to perform core calculations quickly and accurately allows students to extend to more complex mathematical thinking (fluency).
- Engaging with "equal intensity" ensures students will have all the tools they need to apply their knowledge to a wide variety of problems.

Example for Applying the Components of Rigor

Fluency:
- Students need to simply know that 7 x 8 is 56.
  - *[What do I need to know?]*

Conceptual Understanding:
- They need to understand why 7 x 8 is 56.
  - *[How do I know it is true?]*

Application:
- They need to know what benefit there is to knowing that 7 x 8 is 56.
  - *[Why do I need to know it?]*

Engaging the Three Components of Rigor

- Purpose: to develop an understanding of Rigor
- Materials: "Components of Rigor" worksheet
- Directions:

1) Individually, check the components of rigor likely to be required in a task that targets the standard. Make notes about your rationale.
2) Discuss, using these questions to guide you:

- *What makes you think a particular component of rigor applies?*
- *Are there certain words or phrases in the standard that serve as clues?*

Activity


Questions & Comments
COLLEGE & CAREER READINESS STANDARDS FOR ADULT EDUCATION
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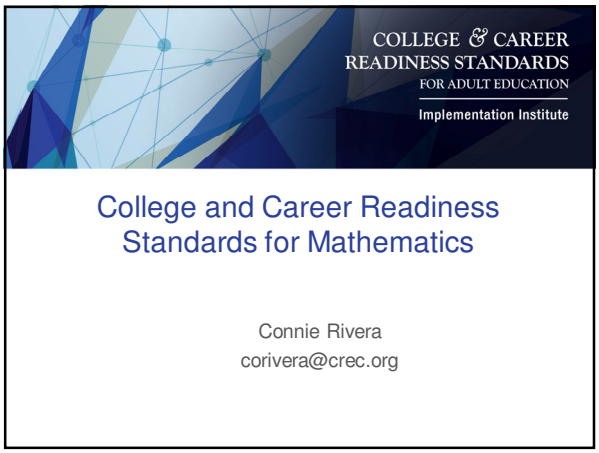
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